USN | | | | | | | | | | MATDIP401

Fourth Semester B.E. Degree Examination, June/July 2016
Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find the angle between any two diagonals of a cube. **(07 Marks)**

b. Prove that the general equation of first degree in x, y, z represents a plane. **(07 Marks)**

c. Find the angle between the lines,

$\frac{x-1}{1}=\frac{y-5}{0}=\frac{z+1}{5}$ and $\frac{x+3}{3}=\frac{y}{5}=\frac{z-5}{2}$. **(06 Marks)**

2 a. Prove that the lines,

$\frac{x-5}{3}=\frac{y-1}{1}=\frac{z-5}{-2}$ and $\frac{x+3}{1}=\frac{y-5}{3}=\frac{z}{5}$ are perpendicular. **(07 Marks)**

b. Find the shortest distance between the lines.

$\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$. **(07 Marks)**

c. Find the equation of the plane containing the point (2, 1, 1) and the line, $\frac{x+1}{2}=\frac{y-2}{3}=\frac{z+1}{-1}$ **(06 Marks)**

3 a. Find the constant 'a' so that the vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}+2\hat{j}-3\hat{k}$ and $3\hat{i}+a\hat{j}+5\hat{k}$ are co-planar. **(07 Marks)**

b. If $\vec{a}=2\hat{i}+3\hat{j}-4\hat{k}$ and $\vec{b}=8\hat{i}-4\hat{j}+\hat{k}$ then prove that $\vec{a}$ is perpendicular to $\vec{b}$ and also find $\left|\vec{a}\times\vec{b}\right|$. **(07 Marks)**

c. Find the volume of the parallelopiped whose co-terminal edges are represented by the vectors,

$\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$ **(06 Marks)**

4 a. Find the velocity and acceleration of a particle moves along the curve $\vec{r}=e^{-2t}\hat{i}+2\cos 5t\hat{j}+5\sin 2t\hat{k}$ at any time 't'. **(07 Marks)**

b. Find the directional derivative of x^2yz^3 at (1, 1, 1) in the direction of $\hat{i}+\hat{j}+2\hat{k}$. **(07 Marks)**

c. Find the divergence of the vector $\vec{F}=(xyz+y^2z)\hat{i}+(3x^2+y^2z)\hat{j}+(xz^2-y^2z)\hat{k}$. **(06 Marks)**

5 a. $\vec{F}=(x+y+1)\hat{i}+\hat{j}-(x+y)\hat{k}$, show that $\vec{F}\cdot \operatorname{curl}\vec{F}=0$. **(07 Marks)**

b. Show that the vector field, $\vec{F}=(3x+3y+4z)\hat{i}+(x-2y+3z)\hat{j}+(3x+2y-z)\hat{k}$ is solenoidal. **(07 Marks)**

c. Find the constants a, b, c such that the vector field,

$\vec{F}=(x+y+az)\hat{i}+(x+cy+2z)\hat{j}+(bx+2y-z)\hat{k}$ is irrotational. **(06 Marks)**

6 a. Prove that $L(\sin at) = \dfrac{a}{s^2 + a^2}$. (07 Marks)

b. Find $L[\sin t \quad \sin 2t \quad \sin 3t]$. (07 Marks)

c. Find $L[\cos^3 t]$. (06 Marks)

7 a. Find the inverse Laplace transform of $\dfrac{1}{(s+1)(s+2)(s+3)}$. (07 Marks)

b. Find $L^{-1}\left[\log\left(1+\dfrac{a^2}{s^2}\right)\right]$. (07 Marks)

c. Find $L^{-1}\left[\dfrac{s+2}{s^2-4s+13}\right]$. (06 Marks)

8 a. Solve the differential equation, $y'' + 2y' + y = 6te^{-t}$ under the conditions $y(0) = 0 = y'(0)$ by Laplace transform techniques. (10 Marks)

b. Solve the differential equation, $y'' - 3y' + 2y = 0$ $y(0) = 0,$ $y'(0) = 1$ by Laplace transform techniques. (10 Marks)

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